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A Method For Estimating Target Acceleration And Its Use In A Missile Guidance Law

by
Richard A. Bednar
Weapons Department

NOVEMBER 1977

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FOREWORD

In this report a method for estimating target velocity and acceleration is presented. A guidance law which uses the estimated target acceleration is implemented in a simplified missile simulation and results are compared with a similar model using proportional navigation.

The method of estimating target acceleration has application to radar guided missiles equipped with inertial sensors, since it requires the measurement of range, range rate, line-of-sight angle and rate as well as missile velocity.

This report is released at the working level. Because of continuing research and study, further modifications may be made to this work.

This effort, accomplished during the period June to August 1976, was funded by AirTask 03P/03P2/008B/3F-32-311-000.

This report has been technically reviewed by A. J. Rice.

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29 November 1977

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Released for publication by
R. M. HILLYER
Technical Director (Acting)

NWC Technical Publication 5913

Published by	Technical Information Department
Collation	Cover and 24 leaves
First printing	60 unnumbered copies

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (14) NWC-TP-5913	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER (9) rept.
4. TITLE (and Subtitle) (6) A METHOD FOR ESTIMATING TARGET ACCELERATION AND ITS USE IN A MISSILE GUIDANCE LAW.		5. TYPE OF REPORT & PERIOD COVERED Interim Study June ~~to~~ Aug~~ust~~ 1976,
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) Richard A. Bednar		8. CONTRACT OR GRANT NUMBER(s) (16) F32311
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Weapons Center China Lake, CA 93555 (17) WF32311000		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AIRTASK 03P/03P2/008B/ 3F-32-311-000
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Weapons Center China Lake, CA 93555	(11)	12. REPORT DATE Nov~~ember~~ 1977
		13. NUMBER OF PAGES 45
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) (12) 48p.		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution limited to U.S. Government agencies only; test and evaluation; ~~20 January 1977~~. Other requests for this document must be referred to the Naval Weapons Center. 29 NOV. 77.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

guidance
target acceleration estimation
simulation
miss distance

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(U) *A Method for Estimating Target Acceleration and Its Use in a Missile Guidance Law,* by Richard A. Bednar. China Lake, Calif., NWC, November 1977. 45 pp. (NWC TP 5913, publication UNCLASSIFIED.)

(U) This report describes an algorithm for estimating target velocity and acceleration using radar and inertial sensor measurements as inputs. In order to determine the accuracy and benefits of such an estimator, the algorithm is incorporated into a missile guidance law. Through the use of a simulation, terminal miss distances are compared with a similar missile using proportional navigation. Simulation results indicate a reduction in miss distance when such an estimate is used because of the missile's ability to recognize and compensate more quickly for the target maneuver.

CONTENTS

INTRODUCTION

If target acceleration could be measured, it is often thought that it could be advantageously used to guide a missile. However, it is not clear as to how to determine target acceleration, exactly what to do with it once one has it, and what can be gained by having it. In a limited sense, this report tries to answer these questions.

In the recent past significant research[1-13] has appeared in open literature describing terminal guidance laws purporting to be improvements (in the sense of reducing terminal miss distance) over proportional navigation. The derivation of such guidance laws is often based on optimal control theory (see Footnote 4) and makes certain assumptions about missile/target geometry and velocities in order to arrive at a linear model more amenable to analysis. In many cases the "optimal" guidance law consists of proportional navigation plus a term proportional to the target acceleration, for example

$$A_{m_c} = N\left(\dot{R}\dot{\sigma} - \frac{1}{2}a_T\right) \tag{1}$$

[1] U.S. Army Missile Command, Redstone Arsenal. *Optimal Controllers for Homing Missiles*, by G. Willems. Huntsville, Alabama, 11 September 1968. (Redstone Arsenal Report RE-TR-68-15.

[2] U.S. Army Missile Command, Redstone Arsenal. *Optimal Controllers for Homing Missiles with Two Time Constants*, by G. Willems. Huntsville, Alabama, October 1969. (Redstone Arsenal Report RE-TR-69-20.

[3] E. I. Axelband and F. W. Hardy. "Quasi-Optimum Proportional Navigation," in *Inst. Elec. Electron. Eng., Trans., Automat. Contr.*, Vol. AC-15, No. 6 (December 1970).

[4] A. E. Bryson and Y. Ho. *Applied Optimal Control*. Blaisdell Publishing Company, Waltham, Massachusetts, 1969.

[5] G. J. Nazaroff. "An Optimal Terminal Guidance Law," in *Inst. Elec. Electron. Eng., Trans., Automat. Contr.*, Vol. AC-21, No. 3 (June 1976).

[6] J. L. Speyer. "An Adaptive Terminal Guidance Scheme Based on an Exponential Cost Criterion with Application to Homing Missile Guidance," in *Inst. Elec. Electron. Eng., Trans., Automat. Contr.*, Vol. AC-21, No. 3 (June 1976).

[7] The Analytic Sciences Corporation. *Adaptive Control and Guidance for Tactical Missiles, Volumes I and II*. Reading, Mass., 30 June 1970. (Report No. TR170-1.) 1970.

[8] Naval Weapons Center. *Improved AIM-7F Guidance Law Study*, by H. D. Nuffer. China Lake, California, NWC, August 1975. (Working papers.)

[9] Air Force Systems Command, Space System Division. *An Optimum Interception Law with Bounded Control in Presence of Noise*, by N. E. Nahi and D. C. Sworder. AFSC, Washington, D.C., 1967. (Report No. SSD-TR-67-124.)

[10] L. A. Stockum and F. C. Weimer. "Optimal and Suboptimal Guidance for a Short-Range Homing Missile," *in Inst. Elec. Electron. Eng., Trans., Aerospace Electron. Syst.*, Vol. AES-12, No. 3 (May 1976).

[11] R. G. Cottrell. "Optimal Intercept Guidance for Short-Range Tactical Missiles," *AIAA Journal*, Vol. 9, No. 7 (July 1971) pp 1414-1415.

[12] R. W. Rishel. "Optimal Terminal Guidance of an Air-to-Surface Missile,' *J. Spacecraft*, Volume 5, No. 6 June 1968.

[13] S. M. Brainin and R. B. McGhee. "Optimal Biased Proportional Navigation," in *Inst. Elec. Electron. Eng., Trans., Automat. Contr.*, Vol. AC-13, No. 4 (August 1968).

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where

A_{m_c} = commanded missile acceleration

N = navigation ratio

$\dot{\sigma}$ = missile/target line of sight rate

a_T = target acceleration

$\dot{R}$ = closing velocity

Such a guidance law is referred to in Footnotes 1, 2, 7 and 8. While the derivation of Equation 1 can be mathematically derived in a routine fashion without physical insight, it is also intuitively appealing. This can be seen by studying Figure 1 which shows a target and missile in the horizontal plane in two different positions (Figure 1a and b). In both figures the target begins a hard turn at time $t = t_1$. If the missile is at the position denoted by $t = t_1$ in Figure 1a, a change of line of sight, $\Delta\sigma_1$, will be observed in a short time Δt. On the other hand if the target is already at the position denoted by $t = t_3$ (Figure 1b) a much larger line-of-sight angle, $\Delta\sigma_2$, is observed in a similar time interval Δt. Noting that proportional navigation uses $\dot{\sigma} = \lim_{\Delta t \to 0} \frac{\Delta\sigma}{\Delta t}$ as its means of guidance, it is seen that effectively the target maneuver is not fully detected until the target has maneuvered for some period of time. The use of the target acceleration term a_T in Equation 1 causes the missile to make a guidance correction as soon as the maneuver starts. As the simulation results presented later in this report indicate, this results in a more direct (shorter length) path between the target and missile and a corresponding shorter time of missile flight than that obtained using proportional navigation.

In order to use the guidance law given by Equation 1, it is necessary to know the target acceleration, a_T. The method most commonly suggested is to use a Kalman filter. In this approach the target is assumed to be undergoing continuous random maneuvers. It is necessary to have an estimate of the target maneuver statistics in order to apply this technique. In addition, use of the basic Kalman

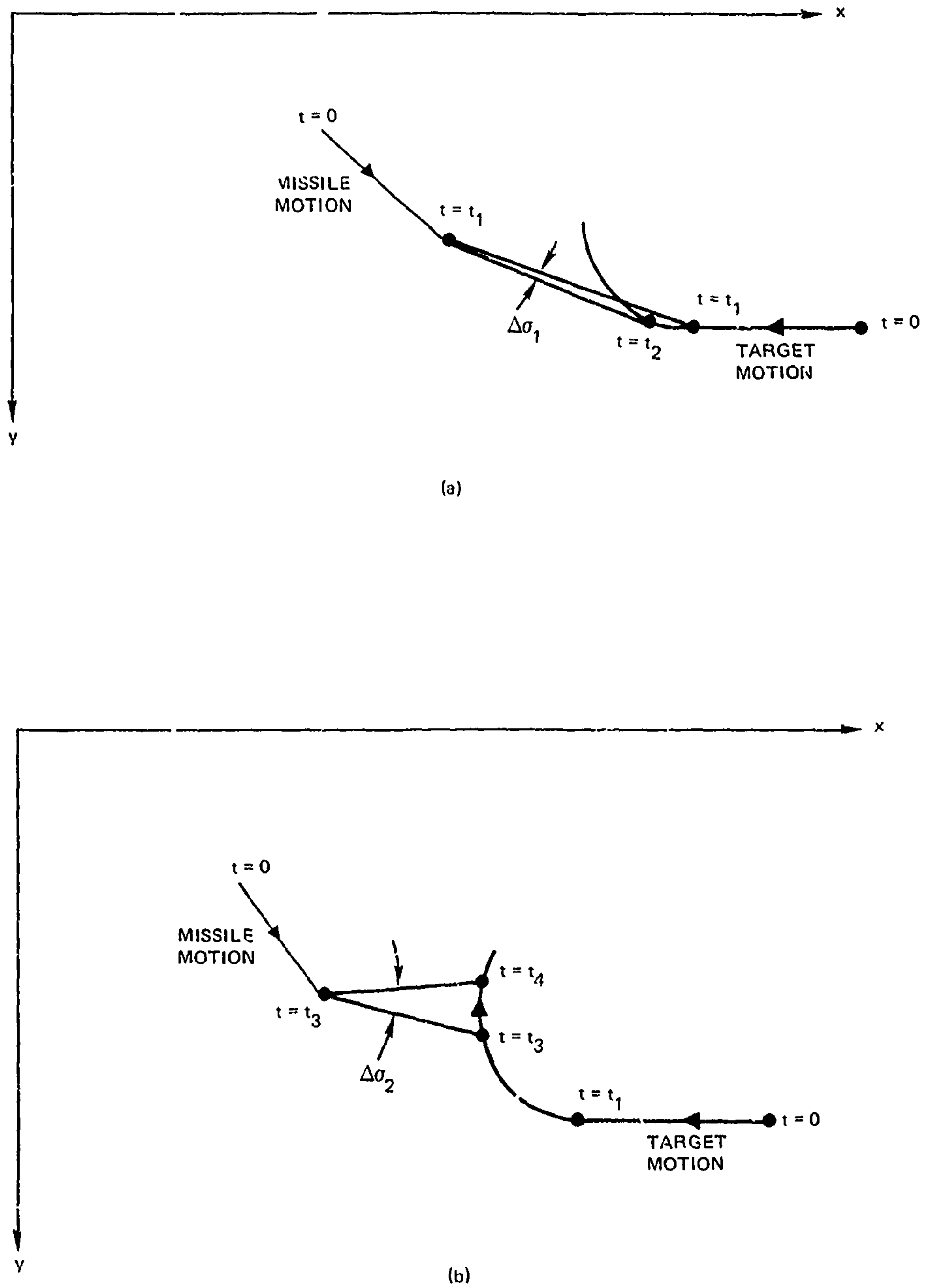


FIGURE 1. Missile/Target Line-of-Sight Angles $\Delta\sigma_1$ and $\Delta\sigma_2$ at Two Different Times.

filter algorithm assumes a linear model of the missile/target system which implies small angle approximations in the kinematic equations. Another approach to estimating the target acceleration which also assumes small angular changes in the missile/target line of sight is to relate the target acceleration to missile acceleration, line-of-sight rate, and acceleration (see Footnote 7)

$$a_T \doteq R\ddot{\sigma} - 2\dot{R}\dot{\sigma} + a_m \tag{2}$$

where

R = range

a_m = missile normal acceleration

$\ddot{\sigma}$ = line-of-sight acceleration

While R, $\dot{R}$, $\dot{\sigma}$ and a_m can be readily measured by existing sensors in missiles, $\ddot{\sigma}$ is more difficult to obtain. Since $\dot{\sigma}$ can be expected to be noisy in the actual missile hardware, differentiation of $\dot{\sigma}$ to obtain $\ddot{\sigma}$ is undesirable. A different approach to estimating target acceleration is presented in this report. It is not necessary to make small angle approximations or to have knowledge of target statistics but rather that the target maneuver can be approximated by an arc of a circle whose radius is changing with time.

In the remaining sections of this report the target velocity and acceleration estimator is derived and analyzed after implementing it in a missile simulation. A guidance law similar to that given by Equation 1 is used to compare trajectories, time histories, and miss distances with those obtained by using proportional navigation.

DERIVATION OF TARGET VELOCITY AND ACCELERATION ESTIMATOR

In this section of the report the basic algorithm for estimating target velocity and acceleration is presented. Only the case when the target and missile motion is constrained to a single plane is considered here. The basic assumptions in the derivation are that the target speed is constant and that the target is either not maneuvering or is turning in a circular arc. Simulation results are presented later in this report when these two assumptions are not met.

Figure 2 shows the trajectories of the missile and target in a common plane taken to be the horizontal inertial plane. As the missile moves from the position denoted by $(x_m(t_1), y_m(t_1))$ to $(x_m(t_2), y_m(t_2))$, the target moves on the circle from the position denoted by $\theta(t_1)$ to that denoted by $\theta(t_2)$. It may turn out that the target is not maneuvering, i.e., the trajectory is a straight line like that shown between times $t = 0$ and $t = t_0$ in Figure 2. However, this can also be thought of as a circle; the radius in this case being infinite.

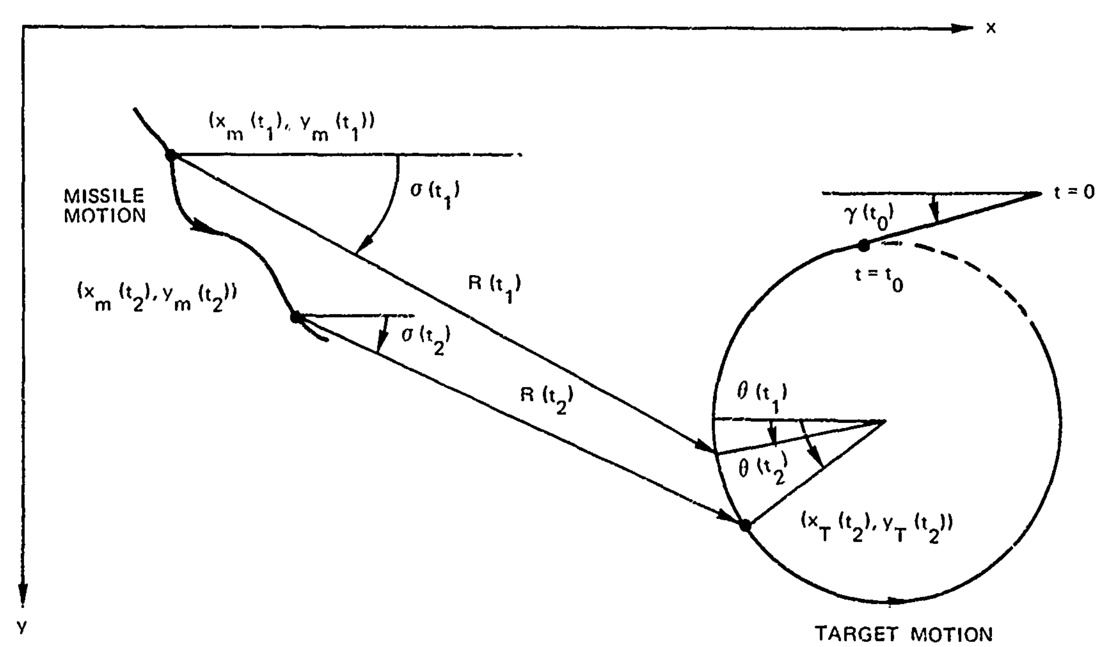


FIGURE 2. Missile/Target Kinematics Used in Estimator Derivation.

In terms of the missile and target coordinates $(x_m(t), y_m(t))$ and $(x_T(t), y_T(t))$, respectively, the range $R(t)$ between target and missile is given by

$$R(t) = \sqrt{R_x^2(t) + R_y^2(t)} \tag{3}$$

where

$$R_x(t) = x_T(t) - x_m(t) \tag{4}$$

$$R_y(t) = y_T(t) - y_m(t) \tag{5}$$

Evaluating Equation 3 at times t_1 and t_2, we have the following expression

$$R^2(t_2) - R^2(t_1) = R_y^2(t_2) + R_x^2(t_2) - R_y^2(t_1) - R_x^2(t_1) \tag{6}$$

Using the definition of a derivative, we also have

$$R(t)\dot{R}(t) = \frac{d}{dt}\frac{R^2(t)}{2} = \lim_{h \to o} \frac{R^2(t + h) - R^2(t)}{2h} \tag{7}$$

In Equation 6 let $t_2 = t + h$ and $t_1 = t$, and substitute the result into Equation 7.

$$R(t)\dot{R}(t) = \lim_{h \to o} \frac{R_y^2(t + h) - R_y^2(t)}{2h} + \lim_{h \to o} \frac{R_x^2(t + h) - R_x(t)}{2h}$$

$$= \frac{d}{dt}\frac{R_y^2(t)}{2} + \frac{d}{dt}\frac{R_x^2(t)}{2}$$

$$= R_y(t)\dot{R}_y(t) + R_x(t)\dot{R}_x(t)$$

Thus

$$R\dot{R} = R_y\dot{R}_y + R_x\dot{R}_x \tag{8}$$

Equation 8 is the first kinematic equation relating target and missile. Another independent equation can be obtained from the line-of-sight angle $\sigma(t)$ as shown in Figure 3.

$$\sigma(t) = \tan^{-1} \frac{R_y(t)}{R_x(t)} \tag{9}$$

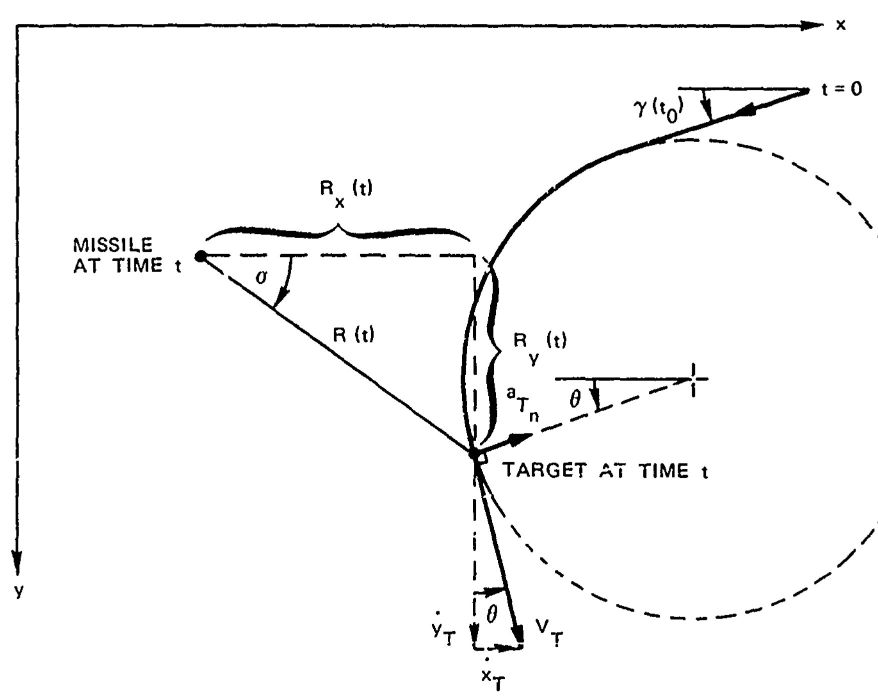


FIGURE 3. Missile/Target Geometry.

Taking the derivative of Equation 9, we have

$$\dot{\sigma} = \frac{1}{1 + (R_y/R_x)^2} \frac{d}{dt}\left(\frac{R_y}{R_x}\right)$$

or

$$R^2\dot{\sigma} = R_x\dot{R}_y - R_y\dot{R}_x \tag{10}$$

Equations 8 and 10 describe the missile and target regardless of the motion of either.

At this point the assumption of circular target motion and constant target speed is used. As indicated in Figure 3, the target normal acceleration a_{T_n} is directed toward the center of the circle and has magnitude

$$a_{T_n} = V_T\dot{\theta} \tag{11}$$

where

V_T = magnitude of target velocity

$\dot{\theta}$ = angular rotation rate of target

a_{T_n} = target acceleration normal to its velocity vector

Integrating Equation 11 between t_1 and t_2, we obtain

$$a_{T_n}(t_2 - t_1) = V_T\left[\theta(t_2) - \theta(t_1)\right]$$

or

$$a_{T_n} = \frac{V_T\left[\theta(t_2) - \theta(t_1)\right]}{t_2 - t_1}, \quad t_1 \neq t_2 \tag{12}$$

where t_1 and t_2 are arbitrary times corresponding to different positions of the target along the circular path.

The x and y components of the target velocity are given by

$$\dot{x}_T(t) = V_T \, \mathrm{sgn}(g_T) \sin \theta(t) \tag{13}$$

$$\dot{y}_T(t) = V_T \, \mathrm{sgn}(g_T) \cos \theta(t) \tag{14}$$

where

$\dot{x}_T(t)$ = x component of target velocity
$\dot{y}_T(t)$ = y component of target velocity
$\theta(t)$ = angular position of target (see either Figure 2 or 3)

$$\mathrm{sgn}(g_T) = \begin{cases} +1 \text{ if } g_T > 0 \\ -1 \text{ if } g_T < 0 \end{cases} \tag{15}$$

$$g_T = a_{T_n}/9.807 \tag{16}$$

The use of $\mathrm{sgn}(g_T)$ is to keep track of the direction of rotation about the circle shown in Figure 2. Positive acceleration ($g_T > 0$) is defined to be in the counterclockwise direction while negative acceleration ($g_T < 0$) is defined to be in the clockwise direction. From Figure 3 we also see that

$$R_x(t) = R \cos \sigma \tag{17}$$

$$R_y(t) = R \sin \sigma \tag{18}$$

Equations 8, 10, 13, 14, 17 and 18 provide the basis for the target velocity and acceleration estimator. Equations 8, 10, 17, and 18 can be combined to give the following equations.

$$\begin{bmatrix} -\sin\sigma & \cos\sigma \\ \cos\sigma & \sin\sigma \end{bmatrix} \begin{bmatrix} \dot{R}_x \\ \dot{R}_y \end{bmatrix} = \begin{bmatrix} R\dot{\sigma} \\ \dot{R} \end{bmatrix} \tag{19}$$

Inverting and solving for $\dot{R}_x$ and $\dot{R}_y$, we have

$$\dot{R}_x = -R\dot{\sigma}\sin\sigma + \dot{R}\cos\sigma \tag{20}$$

$$\dot{R}_y = R\dot{\sigma}\cos\sigma + \dot{R}\sin\sigma \tag{21}$$

By definition

$$\dot{R}_x = \dot{x}_T - \dot{x}_m \tag{22}$$

$$\dot{R}_y = \dot{y}_T - \dot{y}_m \tag{23}$$

Rewriting Equations 13 and 14 and using Equations 20 through 23, we have the following:

$$\dot{x}_T = V_T\,\mathrm{sgn}(g_T)\sin\theta = \dot{x}_m + \dot{R}\cos\sigma - R\dot{\sigma}\sin\sigma \tag{24}$$

$$\dot{y}_T = V_T\,\mathrm{sgn}(g_T)\cos\theta = \dot{y}_m + \dot{R}\sin\sigma + R\dot{\sigma}\cos\sigma \tag{25}$$

The magnitude of the target velocity is then

$$V_T = \sqrt{\dot{x}_T^2 + \dot{y}_T^2} \tag{26}$$

$$= \sqrt{(\dot{x}_m + \dot{R}\cos\sigma - R\dot{\sigma}\sin\sigma)^2 + (\dot{y}_m + \dot{R}\sin\sigma + R\dot{\sigma}\cos\sigma)^2}$$

and the angular position (see Figure 3) of the target is given by

$$\tan\theta = \frac{V_T \sin\theta}{V_T \cos\theta} = \frac{V_T\, \mathrm{sgn}(g_T) \sin\theta}{V_T\, \mathrm{sgn}(g_T) \cos\theta} \tag{27}$$

Substituting Equations 24 and 25 into Equation 27 gives

$$\tan\theta = \frac{\dot{x}_T}{\dot{y}_T} = \frac{\dot{x}_m + \dot{R}\cos\sigma - R\dot{\sigma}\sin\sigma}{\dot{y}_m + \dot{R}\sin\sigma + R\dot{\sigma}\cos\sigma} \tag{28}$$

or

$$\theta(t) = \tan^{-1}\left\{\frac{\dot{x}_m(t) + \dot{R}(t)\cos\sigma(t) - R(t)\dot{\sigma}(t)\sin\sigma(t)}{\dot{y}_m(t) + \dot{R}(t)\sin\sigma(t) + R(t)\dot{\sigma}(t)\cos\sigma(t)}\right\} \tag{29}$$

Once $\theta(t)$ is determined at two different values, t_1 and t_2 of time, they can be substituted along with V_T into Equation 12 to give the target acceleration a_{T_n} normal to its velocity vector.

In many cases it is not necessary to compute the arc tangent as given in Equation 29; an approximation works equally well. If we let

$$\Delta t = t_2 - t_1 = \text{"sampling interval"} \tag{30}$$

we can use Equation 28 to write Equation 12 as

$$a_{T_n} = \frac{V_T\left[\theta(t_2) - \theta(t_1)\right]}{\Delta t}$$

$$= \frac{V_T}{\Delta t}\left[\tan^{-1}\left(\frac{\dot{x}_T(t_2)}{\dot{y}_T(t_2)}\right) - \tan^{-1}\left(\frac{\dot{x}_T(t_1)}{\dot{y}_T(t_1)}\right)\right] \tag{31}$$

or

$$\tan\left(\frac{a_{T_n}\Delta t}{V_T}\right) = \tan\left[\tan^{-1}\left(\frac{\dot{x}_T(t_2)}{\dot{y}_T(t_2)}\right) - \tan^{-1}\left(\frac{\dot{x}_T(t_1)}{\dot{y}_T(t_1)}\right)\right] \tag{32}$$

A trigonometric identity can be applied to the right side of Equation 32 to give the following

$$\tan\left(\frac{a_{T_n}\Delta t}{V_T}\right) = \frac{\tan\left[\tan^{-1}\left(\frac{\dot{x}_T(t_2)}{\dot{y}_T(t_2)}\right)\right] - \tan\left[\tan^{-1}\left(\frac{\dot{x}_T(t_1)}{\dot{y}_T(t_1)}\right)\right]}{1 + \tan\left[\tan^{-1}\left(\frac{\dot{x}_T(t_2)}{\dot{y}_T(t_2)}\right)\right]\tan\left[\tan^{-1}\left(\frac{\dot{x}_T(t_1)}{\dot{y}_T(t_1)}\right)\right]}$$

$$= \frac{\frac{\dot{x}_T(t_2)}{\dot{y}_T(t_2)} - \frac{\dot{x}_T(t_1)}{\dot{y}_T(t_1)}}{1 + \frac{\dot{x}_T(t_2)}{\dot{y}_T(t_2)}\cdot\frac{\dot{x}_T(t_1)}{\dot{y}_T(t_1)}} \tag{33}$$

If we assume that we can replace $\tan\left(\frac{a_{T_n}\Delta t}{V_T}\right)$ by $\frac{a_{T_n}\Delta t}{V_T}$, we have the following

$$a_{T_n} \doteq \frac{V_T}{\Delta t}\left\{\frac{b(t_2) - b(t_1)}{1 + b(t_2)\ b(t_1)}\right\} \tag{34}$$

where

$$b(t) = \frac{\dot{x}_T(t)}{\dot{y}_T(t)} \tag{35}$$

The approximation that

$$\tan\left(\frac{a_{T_n}\,\Delta t}{V_T}\right) \doteq \frac{a_{T_n}\,\Delta t}{V_T} \tag{36}$$

though not necessary, would seem to be realistic for most applications. For example, if the target is pulling 10 g's, (a_{T_n} = 98.07 meters per second squared (m/s^2)), the target velocity is 200 m/s (656 fps) and the acceleration is to be estimated every tenth of a second (Δt = 0.1 sec), we have

$$\frac{a_{T_n}\,\Delta t}{V_T} = 0.049035 \text{ radians}$$

while

$$\tan\left(\frac{a_{T_n}\,\Delta t}{V_T}\right) = 0.049074 \text{ radians}$$

As Equation 36 indicates, the smaller the target acceleration, the shorter the time interval between estimation samples and the higher the target velocity, the better the approximation in Equation 34 will be. All of the simulation results presented later in this report estimate the target acceleration by using Equation 34.

While Equations 31 or 34 can be used to estimate the target acceleration normal to the target velocity vector, it is not this acceleration that is used in the guidance law given by Equation 1 but rather the acceleration perpendicular to the line of sight between target and missile. This component of the normal acceleration can be determined from the geometry shown in Figure 4. The component of target acceleration perpendicular to the line of sight is

$$a_{T_\perp} = \left|a_{T_n}\right| \sin(\sigma + \theta) \tag{37}$$

$a_{T_\perp}$ can thus be determined by using Equation 29 to determine θ and either Equation 31 or 34 to determine a_{T_n} while the line-of-sight angle σ is assumed to be measurable. The purpose of the absolute value of a_{T_n} instead of just a_{T_n} is to account for the direction of rotation of the target and to assign a direction to $a_{T_\perp}$.

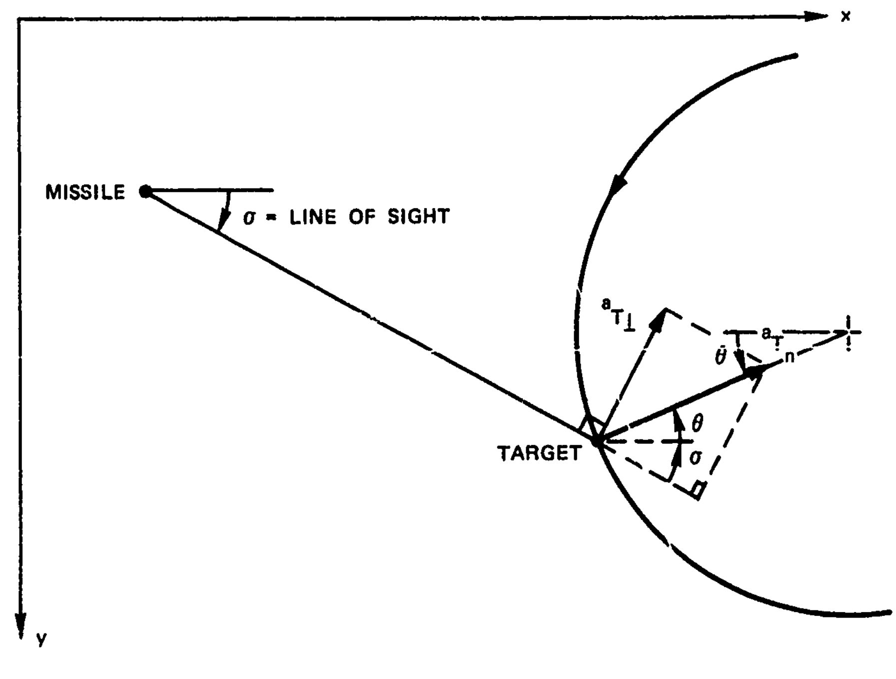


FIGURE 4. Geometry for Determining Target Acceleration $a_{T_\perp}$ Perpendicular to Line of Sight.

An alternate expression for $a_{T_\perp}$ can be obtained by expanding Equation 37:

$$a_{T_\perp} = |a_{T_n}|(\sin\sigma\cos\theta + \sin\theta\cos\sigma) \tag{38}$$

Substituting Equations 13, 14, 17, and 18 into Equation 38, gives

$$\begin{aligned} a_{T_\perp} &= |a_{T_n}|\left(\frac{R_y}{R}\,\frac{\dot{y}_T}{V_T\,\mathrm{sgn}(g_T)} + \frac{\dot{x}_T}{V_T\,\mathrm{sgn}(g_T)}\,\frac{R_x}{R}\right) \\ &= \frac{|a_{T_n}|\ \mathrm{sgn}(a_{T_n})(R_y\dot{y}_T + \dot{x}_T R_x)}{V_T R} \\ &= \frac{a_{T_n}(R_y\dot{y}_T + R_x\dot{x}_T)}{V_T R} \end{aligned} \tag{39}$$

In this form $a_{T_\perp}$ can be obtained without computing the arc tangent shown in Equation 29 as long as the approximation in Equation 36 is accepted. The components of target velocity, $\dot{x}_T$, $\dot{y}_T$, shown in Equation 39 are given by Equations 24 and 25.

SUMMARY OF ESTIMATOR ALGORITHM

The method of determining target velocity and acceleration for a target following a circular path, as derived in the previous section, provides the basis of the general target velocity and acceleration estimator. Using those results, the algorithm shown in Figure 5 is obtained. In this figure the symbol ^ is placed over the estimated variables to distinguish them from the true variables.

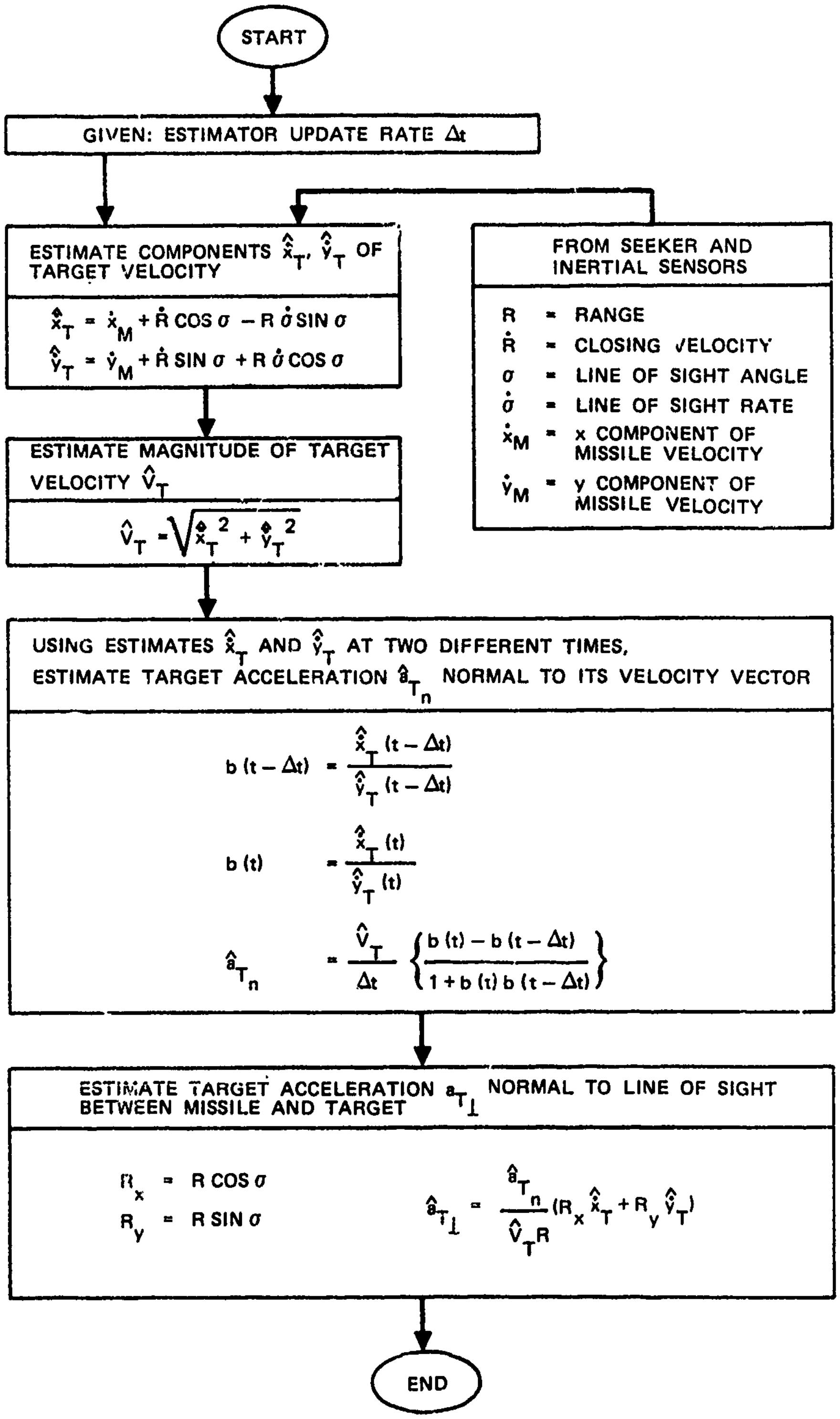


FIGURE 5. Target Velocity and Acceleration Estimator Algorithm.

One of the inputs to the estimator algorithm is the time interval Δt between estimates of the target acceleration. From the point of accuracy it would be desirable for this interval to be small while from a computational burden point it would be desirable for Δt to be large. Simulation results, some of which are presented later in this report, indicate that a value of $\Delta t = 0.1$ sec provides reasonable accuracy in the estimate. Additional inputs to the estimator include range, range rate (closing velocity), line-of-sight angle and rate as well as components of missile velocity. Note that the line-of-sight angle and components of missile velocity are referenced to an inertial system and not to the body of the missile. The components of target velocity, $\dot{x}_T$, $\dot{y}_T$, are then estimated using these inputs and Equations 24 and 25. The magnitude V_T of the target velocity is then obtained from Equation 26.

In order to estimate the target acceleration it is necessary to estimate and store the components of estimated target velocity at two times separated by the sampling interval Δt. Equations 34 and 35 are then used with $t_1 = t - \Delta t$ and $t_2 = t$ to estimate the target acceleration normal to its velocity vector. Finally, Equation 39 is used to estimate the target acceleration normal to the line of sight between target and missile.

EVALUATION OF ESTIMATOR

In order to gain some insight into how well the estimator works and what can be gained in the way of improved missile performance by using such an estimator, a simple model of a missile and target was constructed and simulated on a digital computer. The accuracy of the estimator was observed by comparing estimated values of target velocity and acceleration with exact values. This was done under conditions not satisfying the assumptions under which the estimator was derived as well as under those conditions for which it was derived. Time histories of missile acceleration and missile/target trajectories were compared. The variable in the comparison was the guidance law, i.e., missile performance was compared with and without the target acceleration estimator in the guidance law. The two guidance laws compared were

$$A_{m_c} = N\dot{R}\dot{\sigma} \qquad \text{(proportional navigation)} \tag{40}$$

and

$$A_{m_c} = N\left(\dot{R}\dot{\sigma} - \frac{1}{2}S(t_{go})\hat{a}_{T_\perp}\right) \tag{41}$$

where

A_{m_c} = commanded missile acceleration measured in the wind axis system

N = navigation ratio = 3

$\dot{\sigma}$ = missile/target line-of-sight rate

$\hat{a}_{T_\perp}$ = estimated target acceleration perpendicular to the missile/target line of sight

$\dot{R}$ = closing velocity

$$S(t_{go}) = \begin{cases} 1 \text{ if } t_{go} \geq t_{go_{min}} \text{ sec} \\ 0 \text{ if } t_{go} < t_{go_{min}} \text{ sec} \end{cases} \tag{42}$$

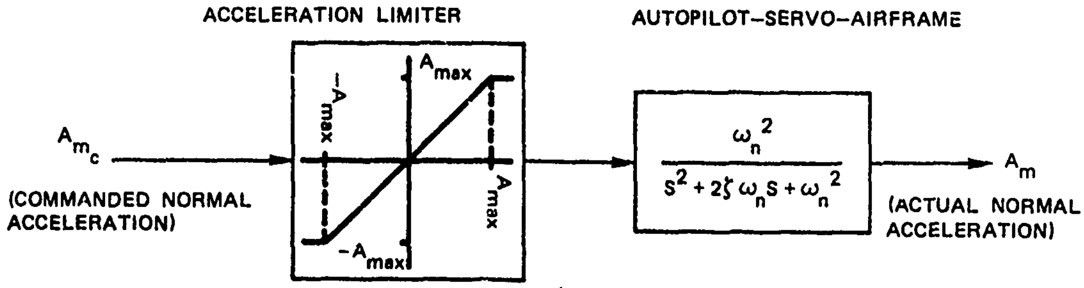
$$t_{go} = R/\dot{R} \text{ (approximate "time to go" until intercept)} \tag{43}$$

This is essentially a comparison of proportional navigation with proportional navigation biased by the estimated target acceleration. A comparison of Equations 1 and 41 reveals the addition of a switch $S(t_{go})$ in Equation 41 which switches the estimator out of the guidance law when the approximate "time-to-go" until intercept, t_{go}, is less than some small value $t_{go_{min}}$. The use of this switch is included because in some cases the estimate of target acceleration became poor when the time to go until intercept became less than 0.5 sec (see Figure 14 for example). This decrease in accuracy of the estimator is thought to be due to the inherent instability of the kinematic equations as the range R goes to zero. In the simulation examples presented later, values of $t_{go_{min}}$ of 1 and 2 sec were used. In all example trajectories presented below, with the exception of Example 5, the target acceleration estimate $\hat{a}_{T_\perp}$ used in Equation 41 was updated every 0.1 sec. Between updates the target acceleration estimate is held constant at the value of the most recent estimate. The constant 1/2 appearing in Equation 41 results from theoretical guidance law studies (see Footnotes 1, 7, and 8) based on simplifying assumptions. No attempt was made to optimize this parameter. Its value could be determined by a Monte-Carlo study.

SIMULATION MODEL

The missile and target were modeled as point masses. Only motion in a horizontal plane was considered. A constant missile velocity of 1 kilometer per second (.54 nautical mile per second) was assumed. Target motion varied from one missile/target encounter to the next as described in the example computer simulations described below. The commanded missile accelerations as given by Equations 40 or 41 were applied to an acceleration command limiter and second order airframe model as shown in Figure 6. Numerical values of the parameters appearing in Figure 6 correspond to a missile currently under investigation and are discussed further in the examples shown below. Missile acceleration A_m is converted from wind axes[14] to inertial coordinates and integrated to give velocity. Since the magnitude of the missile velocity will tend to increase slightly due to numerical integration errors, the magnitude of the velocity is normalized to the assumed constant value of 1 kilometer per second (.54 nautical mile per second). The resultant components of missile velocity are then integrated to provide missile position so that the relative position of the missile and target can be computed.

[14] B. Etkin. *Dynamics of Flight.* John Wiley and Sons, New York, 1959.



ω_n = AUTOPILOT–SERVO–AIRFRAME NATURAL FREQ (RAD/SEC)
ζ = AUTOPILOT–SERVO–AIRFRAME DAMPING RATIO = 0.707
S = LaPLACE TRANSFORM VARIABLE

FIGURE 6. Missile Pitch Plane Model.

It is noted that the use of the target acceleration estimator given in this report is not restricted to or derived on the basis of this simplified model, but rather it is being used as a simple tool to study its performance. On the following pages a number of example missile/target encounters which compare missile performance with and without the target acceleration estimator are simulated and discussed.

Example 1. In this example the missile and target are initially separated by 22 kilometers (11.9 nautical miles) in a head-on situation. Time t = 0 sec is taken to be the time of the beginning of the endgame encounter. Numerical values of the missile parameters shown in Figure 6 are given in Table 1. As shown in Figure 7, for 6 sec after the beginning of the terminal encounter the target follows a straight line trajectory and then begins a 5-*g* turn in the counterclockwise direction. Figure 7 compares the trajectories of the missile with the use of the target estimator and without it (proportional navigation). As can be seen from this figure, the use of the target acceleration estimator provides a more direct or shorter trajectory to the target due to its ability to compensate for the maneuver. Both missiles guided successfully to the target, however. Figure 8 compares exact missile velocity and acceleration with those computed using the estimation technique given in this report. The quality of the estimation is seen

TABLE 1. Missile and Target Simulation Parameters.

Parameters	Example 1	2	3	4	5
Missile					
Airframe natural frequency ω_n, sec^{-1}	6.4	1.28	6.4	6.4	6.4
Airframe damping ratio, ζ	0.707	0.707	0.707	0.707	0.707
Velocity, meter/second	1000.0	1000.0	1000.0	1000.0	1000.0
Altitude, kilometer	6.0	23.0	6.0	6.0	6.0
Maximum acceleration (A_{max} in Figure 6)	15.0	5.3	15.0	15.0	15.0
Estimator update interval Δt, second	0.1	0.1	0.1	0.1	0.01
Target					
Velocity, meter/second	379.0 (Mach 1.2)	1036.0 (Mach 3.5)	379.0 (Mach 1.2)	Sinusoidal	Sinusoidal
Altitude, kilometer	6.0	23.0	6.0	6.0	6.0
Maneuver magnitude, g's	5.0	2.0	5.0	Variable	Variable
Maneuver initiation, second	6.0	Variable	6.0	0	0
Maneuver time constant, second	0	0	1.0	...	...
Range from missile at launch, kilometer	22.0	28.0	22.0	22.0	22.0

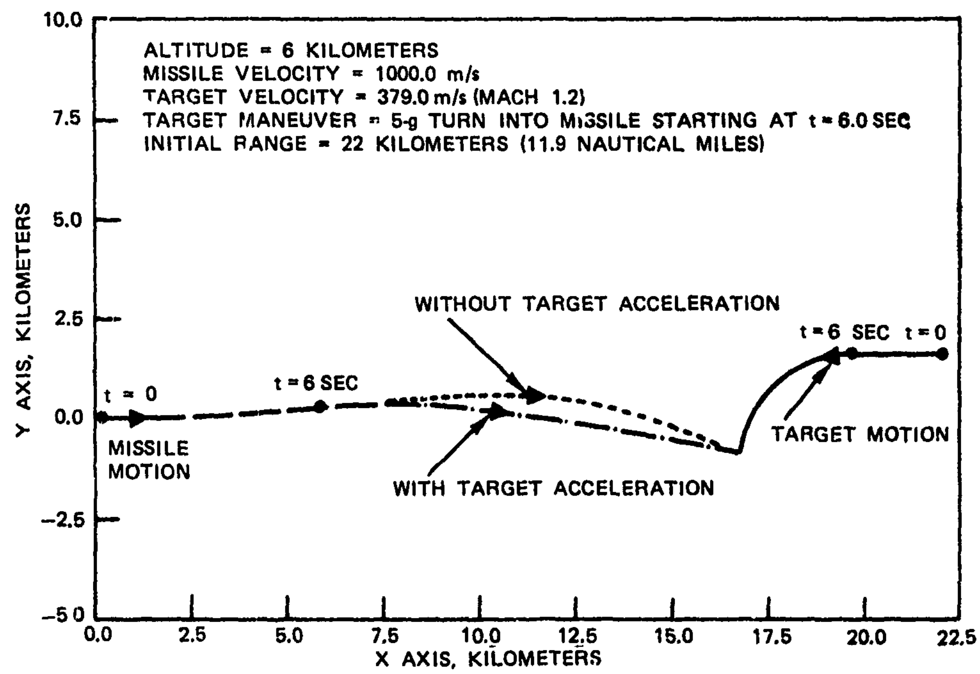


FIGURE 7. Missile and Target Trajectory in the Horizontal Plane for Example 1.

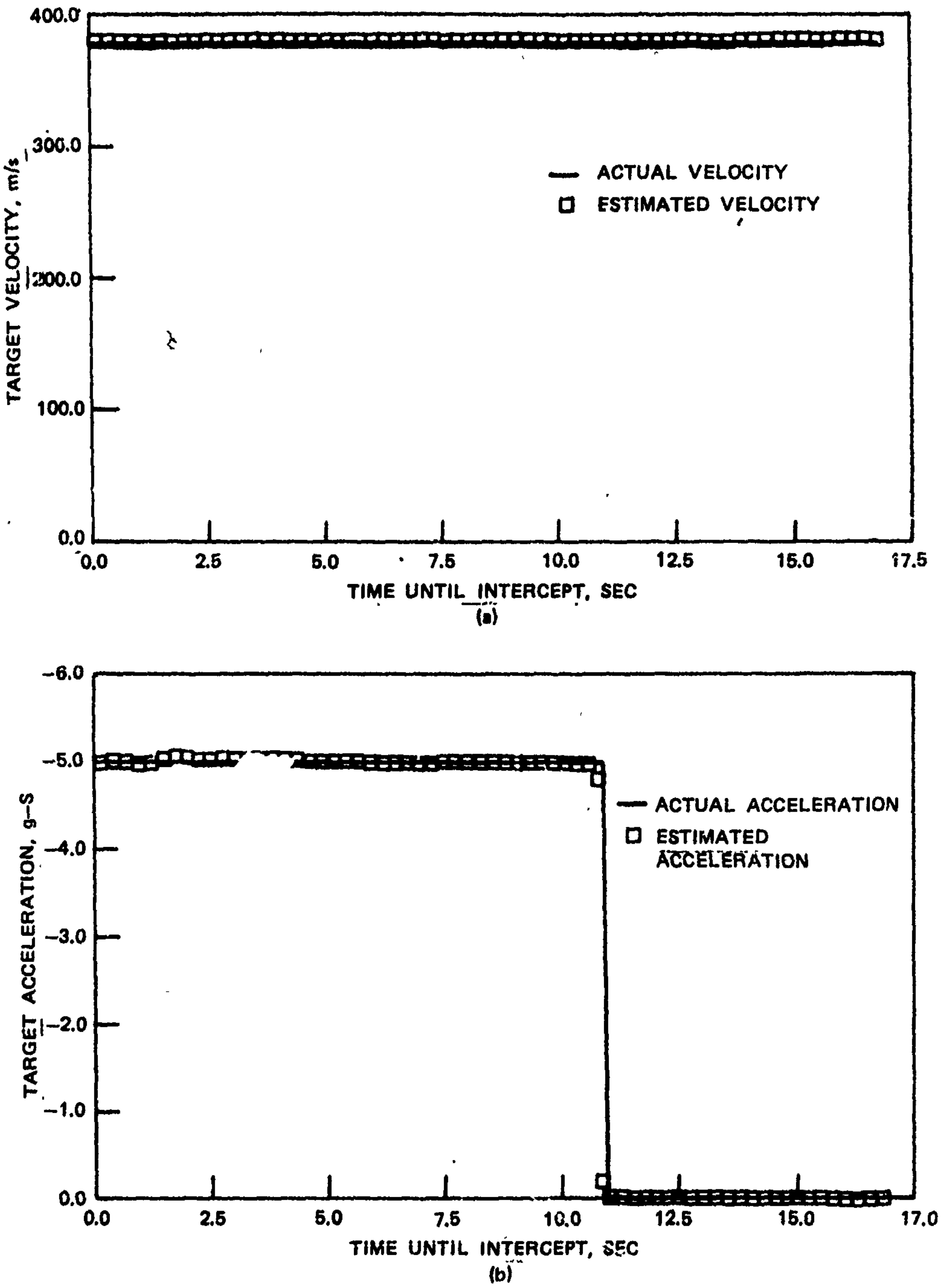
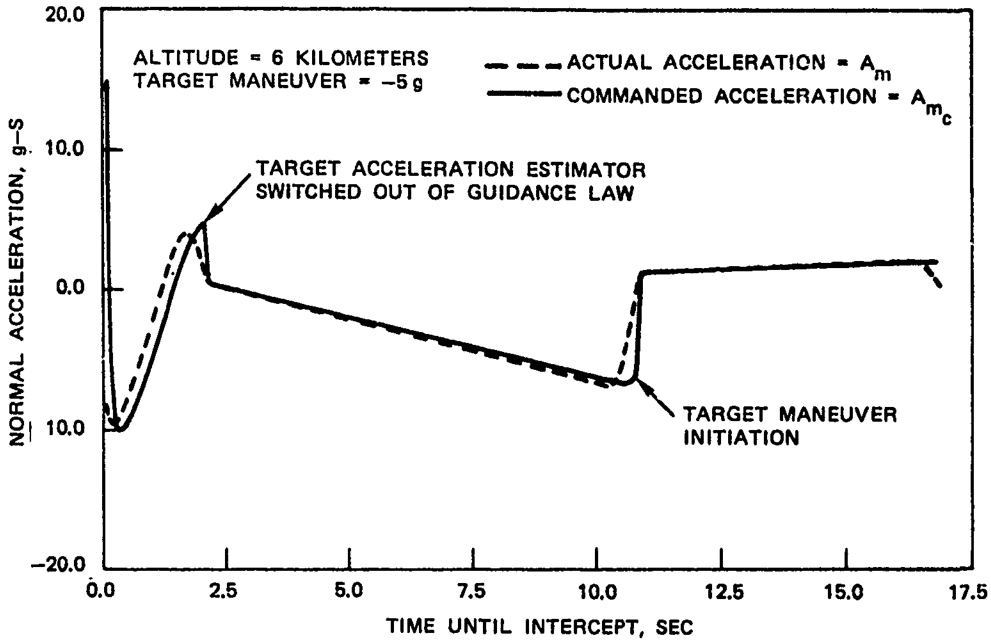
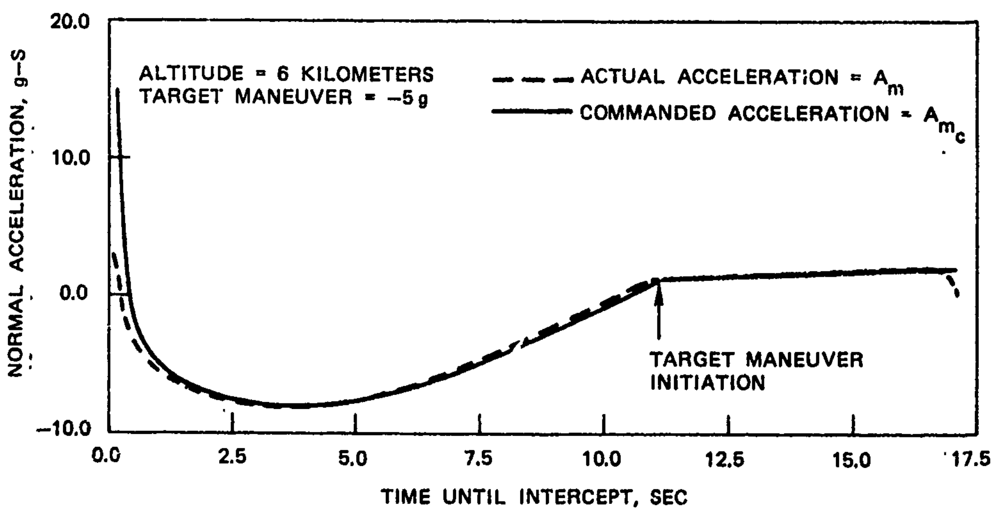


FIGURE 8. Comparison Between Estimated and Actual Target Acceleration and Velocity for Example 1.



(a) WITH TARGET ACCELERATION



(b) WITHOUT TARGET ACCELERATION (PROPORTIONAL NAVIGATION)

FIGURE 9. Missile Acceleration Profiles for Example 1.

to be very good. Figure 9 compares missile acceleration with and without the target acceleration estimator. The acceleration A_m and A_{m_c} shown in this figure refer to those shown in Figure 6. As shown in Figure 9, the target begins to maneuver when the time until intercept is approximately 11 sec.

Comparing missile acceleration profiles with and without the target acceleration estimator show that the missile with the target acceleration estimator effectively reverses its direction approximately 2 sec sooner than the one without the estimator. This ability to detect the maneuver earlier results in its more direct trajectory for the missile with target acceleration estimator as shown in Figure 7. Again referring to Figure 9, an abrupt change in the commanded acceleration occurs in the missile employing the target estimator when the time until intercept is approximately 2 sec. This is due to the estimator being switched out of the guidance law given by Equation 41 when $T_{go_{min}} = 2$ sec in Equation 42.

This example was repeated using a maximum missile lateral acceleration of 5 g instead of 15 g. The missile using proportional navigation missed the target by 440 meters while the missile employing the estimated target acceleration missed by less than 4 meters.

Example 2. The ability of the missile with the target acceleration estimator to more quickly change its course when a maneuver starts and to follow a more direct trajectory as indicated in Example 1 was thought to be of particular advantage at high altitude where missile maneuverability is low and airframe response is slow. In this example, the altitude of missile and target is changed to 23 kilometers. Maximum missile acceleration is limited to 5.3 g's while the target maneuver is taken to be 2 g's as indicated in Table 1. Figure 10 shows the trajectories of target and missiles with and without the target acceleration estimator for $t \geq 6$ sec where $t = 6$ sec is the time at which the maneuver starts. As in Example 1 the use of target acceleration estimation provides a more direct path to the target. In this case the missile without the target acceleration estimator missed the target by 60 meters while the missile with the estimator came within 5 meters of the target. (In all examples considered in this report the simulation was stopped whenever the missile came within 5 meters of the target.) The corresponding missile acceleration time histories are shown in Figure 11. From this figure it can be seen that because the missile without the acceleration estimator is acceleration limited the last 4 sec of flight, it is unable to reach the target. However, the missile with the target acceleration estimator reverses its acceleration sooner after the maneuver starts and is able to guide to the target. Figure 12 shows the comparison between true and estimated target acceleration and velocity; again the agreement is very good.

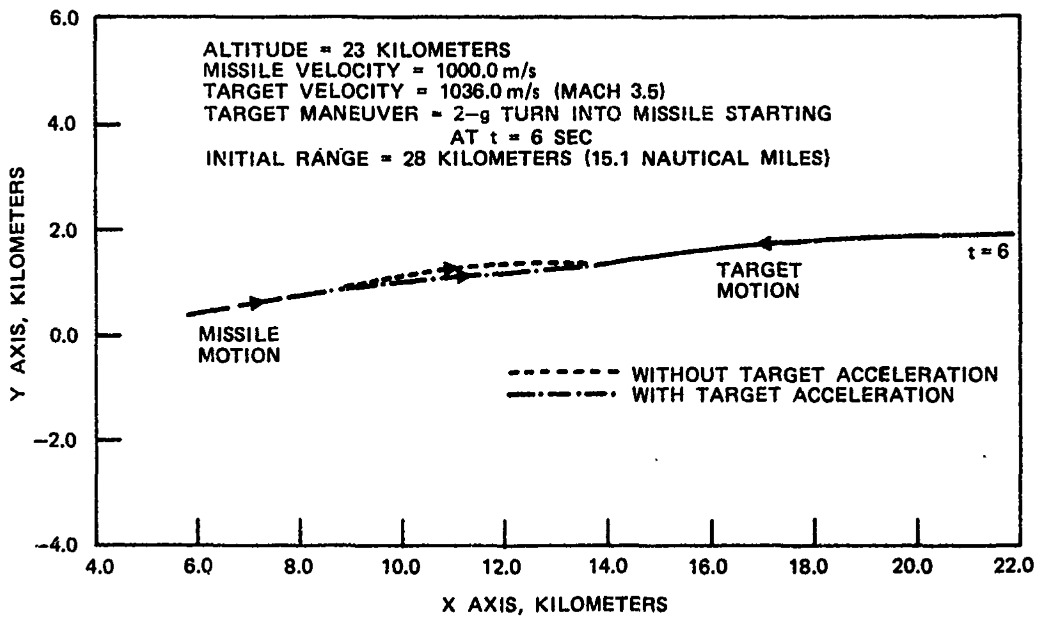


FIGURE 10. Missile and Target Trajectories in the Horizontal Plane for Example 2.

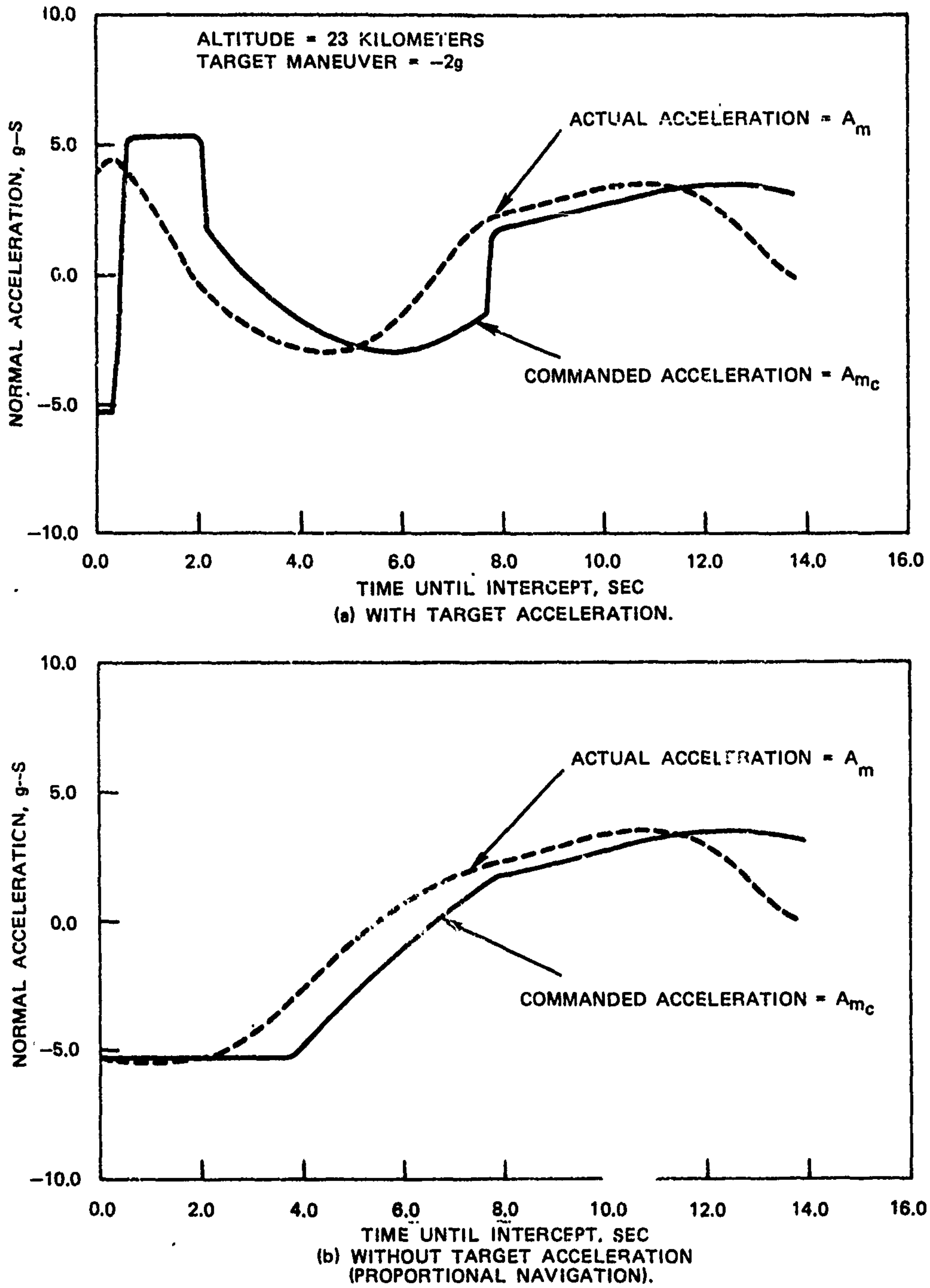


FIGURE 11. Missile Acceleration Profiles for Example 2.

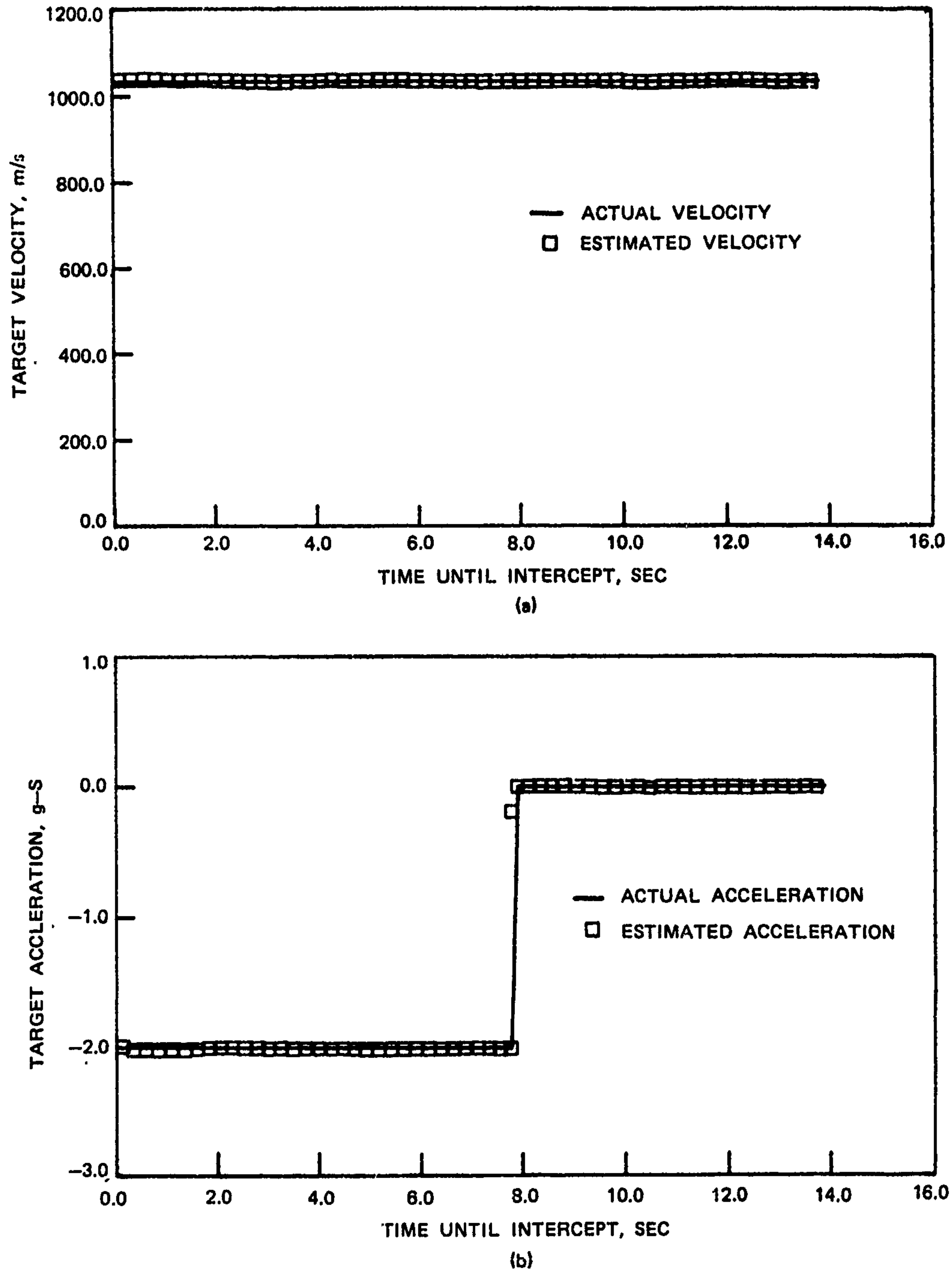


FIGURE 12. Comparison Between Estimated and Actual Target Acceleration and Velocity for Example 2.

The previous paragraph was concerned with a target maneuver starting 6 sec from the beginning of the endgame. In Figure 13 are shown miss distances for target maneuvers of the same acceleration magnitude but starting at different times. As this figure indicates, significant miss distance reduction is possible. In the case when the maneuver starts 12 sec after the terminal encounter begins, the miss distances with and without the target acceleration estimator are the same. This is due to the maneuver being started when the time until intercept is less than 2 sec. In this example, 2 sec until intercept was also chosen to be the time at which the estimator was switched out of the guidance law given by Equation 41. Thus, for the case when the maneuver starts at 12 sec into the endgame, the target acceleration estimator was actually not used and the missile response is the same as if proportional navigation had been used, leading to the same miss distance.

Example 3. In the previous examples it was assumed that the target was moving with constant velocity and instantaneously changed its acceleration from one constant level to another (see Figure 12 for example). These two assumptions are the basic assumptions under which the estimator was derived, and we would expect good results under these conditions. In this example the second assumption is removed, but the first assumption (constant target velocity) is retained. More specifically, the target is switched from 0 to −5 g's starting 6 sec after the endgame begins but changes level exponentially (see Figure 14). Thus the target acceleration is modeled by

$$g_T = g_{T_c}\left(1 - e^{-(t-6)/\tau_T}\right) \text{ for } t \geq 6 \text{ sec} \tag{44}$$

where

g_T = target acceleration (gravity units)

g_{T_c} = commanded target acceleration = −5 g's

τ_T = target time constant = 1 sec

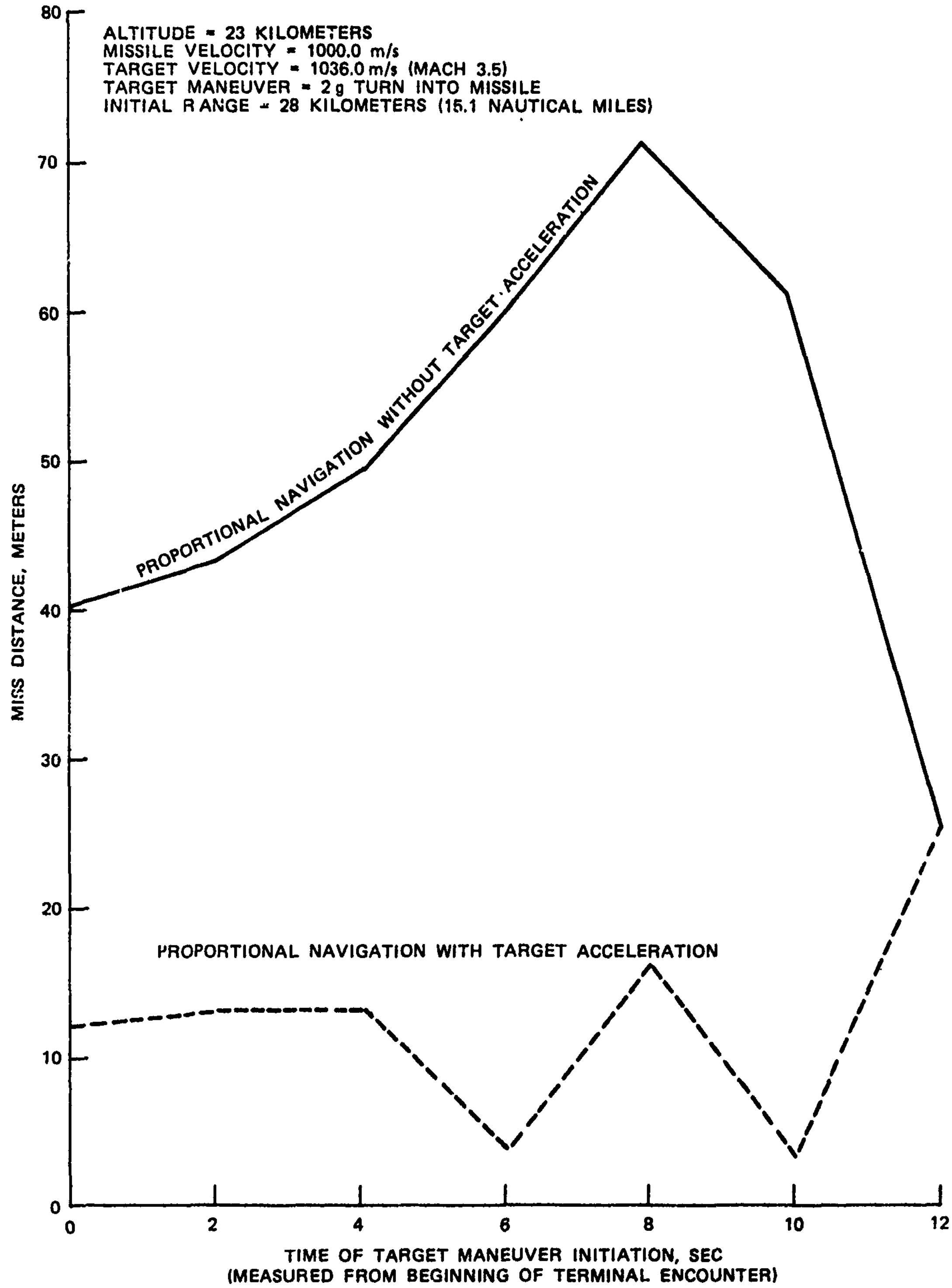
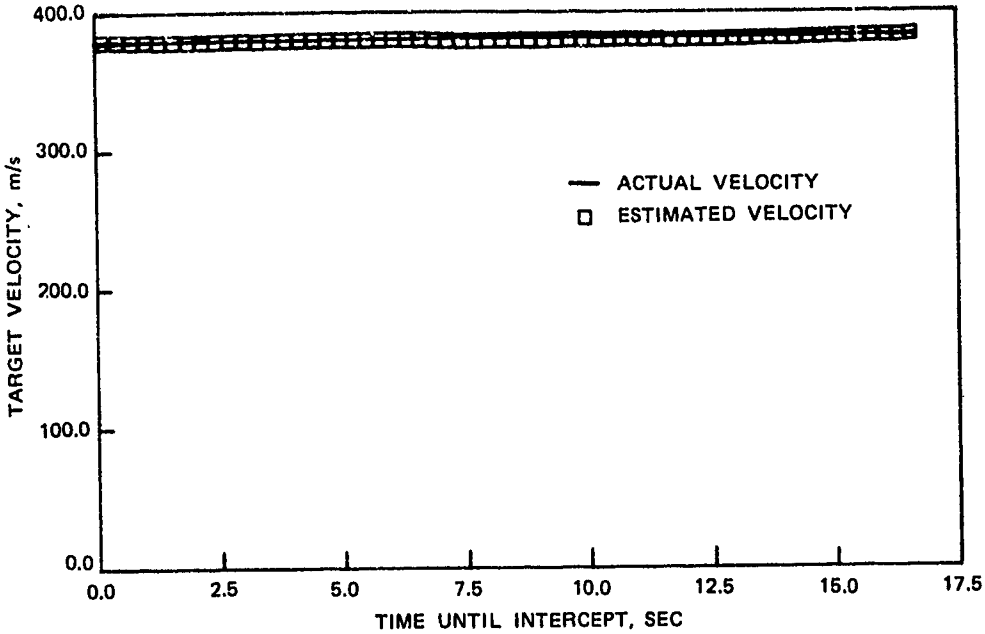
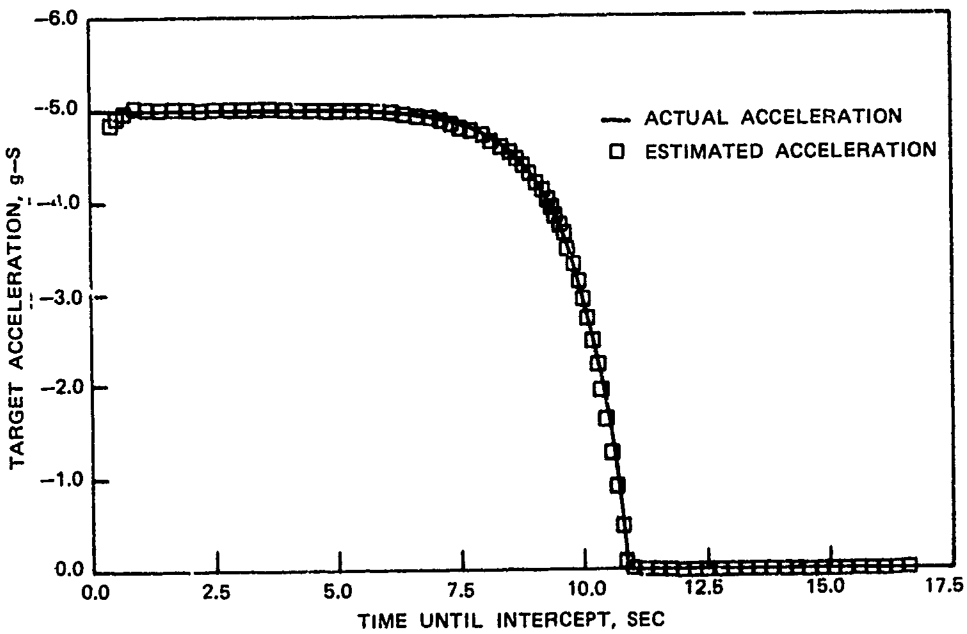


FIGURE 13. Effect on Miss Distance Caused by Including Target Acceleration Estimation in the Guidance Law.



(a)



(b)

FIGURE 14. Comparison Between Estimated and Actual Target Acceleration and Velocity for Example 3.

Simulation parameters are given in Table 1. This example differs from Example 1 only in that the target time constant τ_T is 1 sec in this example and 0 sec in Example 1. Trajectories of the target and the missile with and without the target acceleration estimator are shown in Figure 15. Again, the missile that employs the acceleration estimator follows a more direct (shorter) trajectory to the target than the same missile without the estimator.

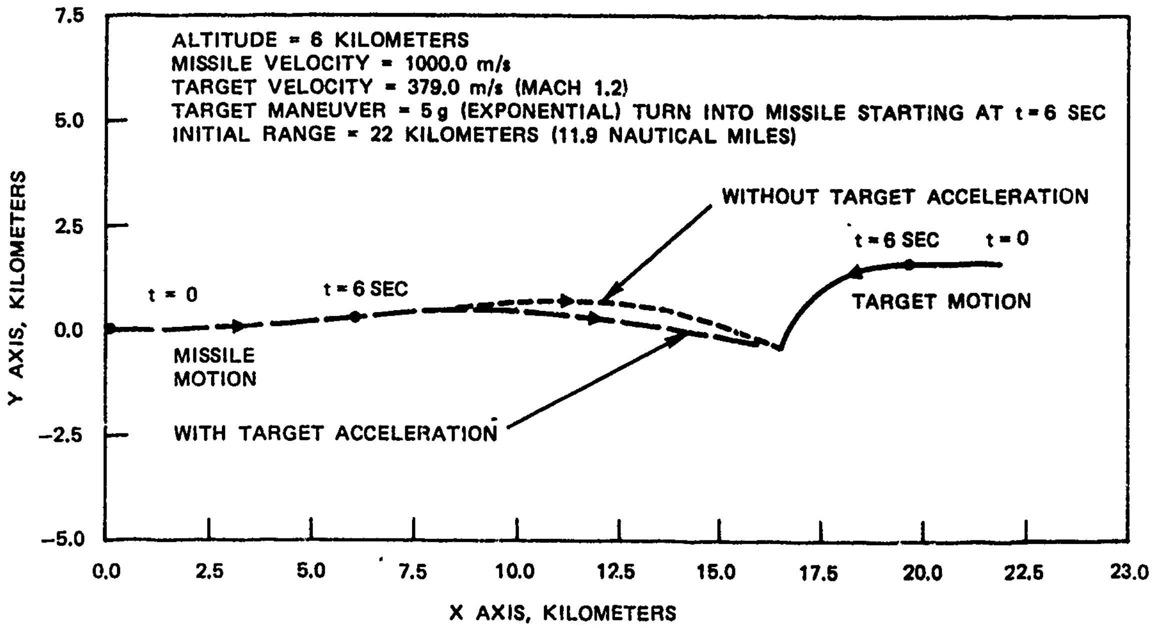


FIGURE 15. Missile and Target Trajectories in the Horizontal Plane for Example 3.

The accuracy of the estimator for this example is indicated in Figure 14. The accuracy of the target velocity is nearly perfect. This follows since the target has constant velocity and this is one of the two assumptions in the derivation of the estimator. Also, the estimator determines the target velocity first and then the target acceleration (not vice versa), so even though the target acceleration is not of the form that was used to derive the estimator the target velocity estimation is not affected. The target acceleration is still good although it tends to lag in time the true value at the time the maneuver begins (see Figure 14 with "time until intercept" = 11 sec). Also, the estimate deteriorates the last fraction of a second. Since the estimator was switched out of the guidance law (Equation 41) when the time until intercept was less than 1 sec, this had no effect on the missile guidance.

Example 4. In this example the target is modeled in such a way that neither of the assumptions used in the derivation of the estimator is satisfied. The coordinates $x_T(t)$, $y_T(t)$ of the target are taken to be

$$x_T(t) = x_T(0) - V_{x_T} t \tag{45}$$

$$y_T(t) = y_{max} \sin \omega_1 t \tag{46}$$

where

V_{x_T} = magnitude of x component of target velocity = 379 meter/sec
y_{max} = maximum y component of target position = 2000.0 meters
ω_1 = frequency of oscillation = 0.24257 rad/sec

Substituting Equation 45 into 46 gives

$$y_T(t) = y_{max} \sin\left[\omega\left(x_T(0) - x_T(t)\right)\right] \tag{47}$$

where

$$\omega = \omega_1/V_{x_T} \tag{48}$$

$$= 0.000640 \text{ rad/meter} \tag{49}$$

Thus, by Equation 47 the target follows a sinusoidal trajectory in the horizontal x,y plane.

Differentiation of Equations 45 and 46 gives

$$\dot{x}_T(t) = -V_{x_T} \tag{50}$$

$$\dot{y}_T(t) = y_{max}\omega V_{x_T} \cos\left[\omega\left(x_T(0) - x_T(t)\right)\right] \tag{51}$$

and

$$V_T = \sqrt{\dot{x}_T^2 + \dot{y}_T^2}$$

$$= V_{x_T}\sqrt{1 + y_{max}^2\omega^2 \cos^2\left[\omega\left(x_T(0) - x_T(t)\right)\right]} \tag{52}$$

where

V_T = target velocity

Thus the target velocity varies between

V_{x_T} = 379 meter/sec

and

$V_{x_T}\sqrt{1 + y_{max}^2\omega^2}$ = 615.6 meters/sec

Using a result from Footnote 15, the target acceleration normal to its path is given by

$$a_{T_n} = \frac{\ddot{y}_T\dot{x}_T - \ddot{x}_T\dot{y}_T}{V_T} \tag{53}$$

[15] D. I. Cook and D. N. Pierce. *Dynamics*, International Textbook Company, 1960, page 13.

Differentiation of Equations 50 and 51 and substitution into Equation 53 gives

$$a_{T_n} = \frac{y_{max}\omega^2 V_{x_T}^2 \sin\left[\omega\left(x(0) - x_T(t)\right)\right]}{\sqrt{1 + y_{max}^2\omega^2 \cos^2\left[\omega\left(x(0) - x_T(t)\right)\right]}} \tag{54}$$

Thus the target normal acceleration varies between

$$\pm y_{max}\omega^2 V_{x_T}^2 = \pm 117.7 \text{ meters/sec}^2 = \pm 12\ g\text{'s}$$

From Equations 52 and 54 it can be seen that the assumptions used to derive the target velocity estimator, i.e., constant velocity and piecewise constant normal acceleration, are not satisfied.

Figure 16 compares the trajectories of target and missile; the latter with and without the target acceleration estimator. As in the other examples, use of the target acceleration estimator provides a more direct trajectory to the target. Figure 17 compares estimated and actual target acceleration and velocity. While the target acceleration estimate is certainly not perfect, the estimated acceleration provides enough bias in the guidance given by Equation 41 to reduce the miss distance from 30 meters using proportional navigation (i.e., no target acceleration estimate) to 8 meters with the target acceleration estimate. Figure 18 compares missile acceleration with and without the target acceleration estimator. As has been observed in many simulation runs, there is a tendency for the missile using the target acceleration estimator to be less *g*-limited (i.e., require less available acceleration normal to its flight path) than the missile using proportional navigation. A discontinuity in the commanded missile acceleration occurs at approximately 1.25 sec until intercept (see Figure 18). This is due to the target acceleration estimator being switched out of the guidance law given by Equation 41 when the estimated time until intercept (as given by Equation 43) is less than 1 sec.

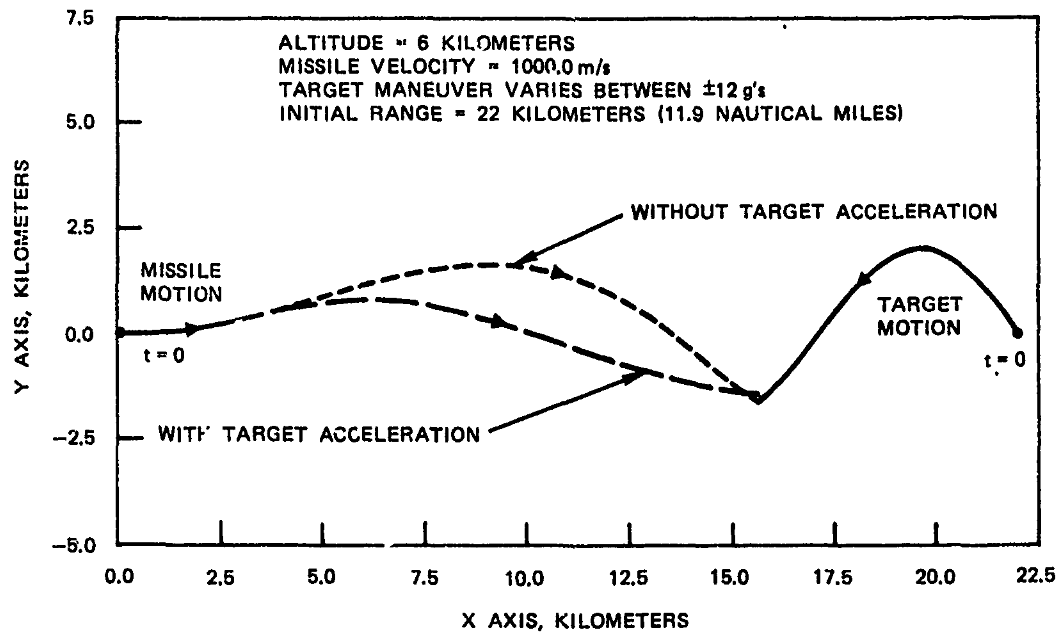


FIGURE 16. Missile and Target Trajectories in the Horizontal Plane for Example 4.

Example 5. This example differs in only one way from Example 4. In Example 4 (as well as all the other previous examples) the time interval between which samples of range, closing velocity, line-of-sight angle, line-of-sight rate, and missile velocity which are used to estimate the target acceleration was taken to be 0.1 sec. In this example the time interval is decreased by a factor of 10 to 0.01 sec to see if the estimated acceleration as given by Figure 17 would be improved by increasing the sampling rate. Figure 19 shows the comparison between estimated and true target acceleration and velocity for the smaller sampling interval (Δt = 0.01 sec). Comparison of these results with those given in Figure 17 which uses the longer sample interval (Δt = 0.1 sec) reveals little improvement. Consequently, the missile trajectory and acceleration time history for this example are essentially the same as those shown in Example 4.

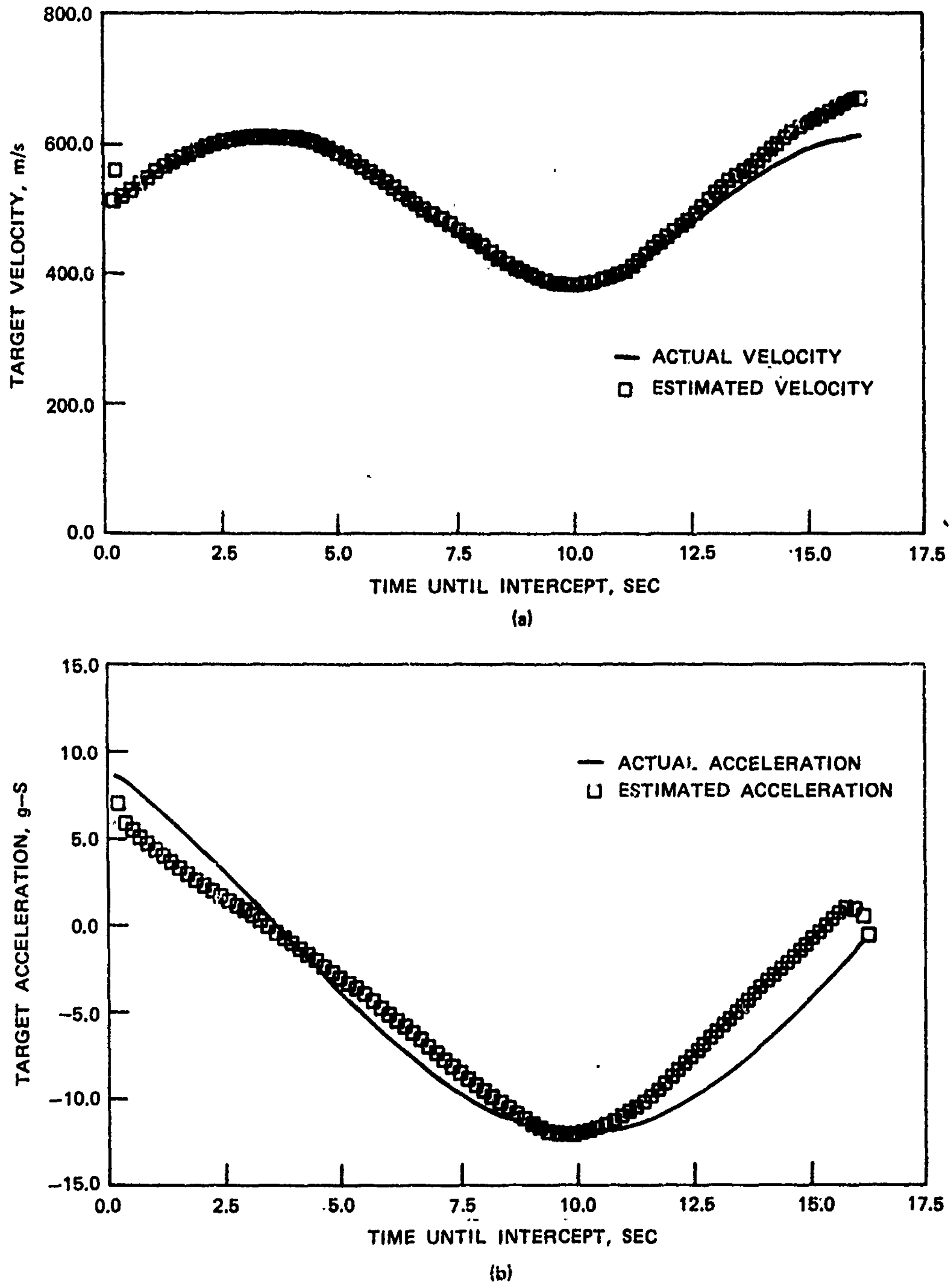


FIGURE 17. Comparison Between Estimated and Actual Target Acceleration and Velocity for Example 4 (Sampling interval $\Delta t = 0.10$ sec).

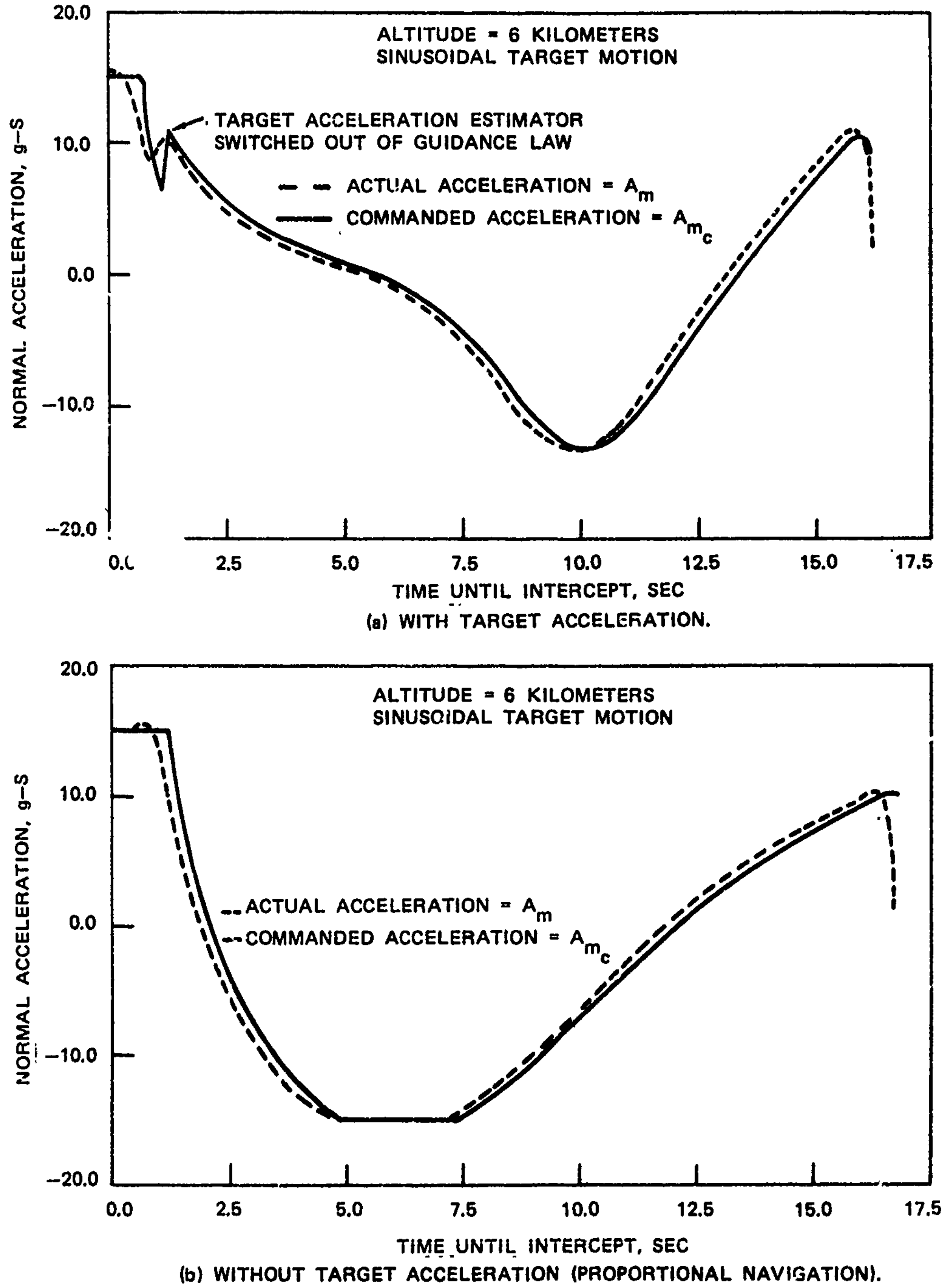


FIGURE 18. Missile Acceleration Profiles for Example 4.

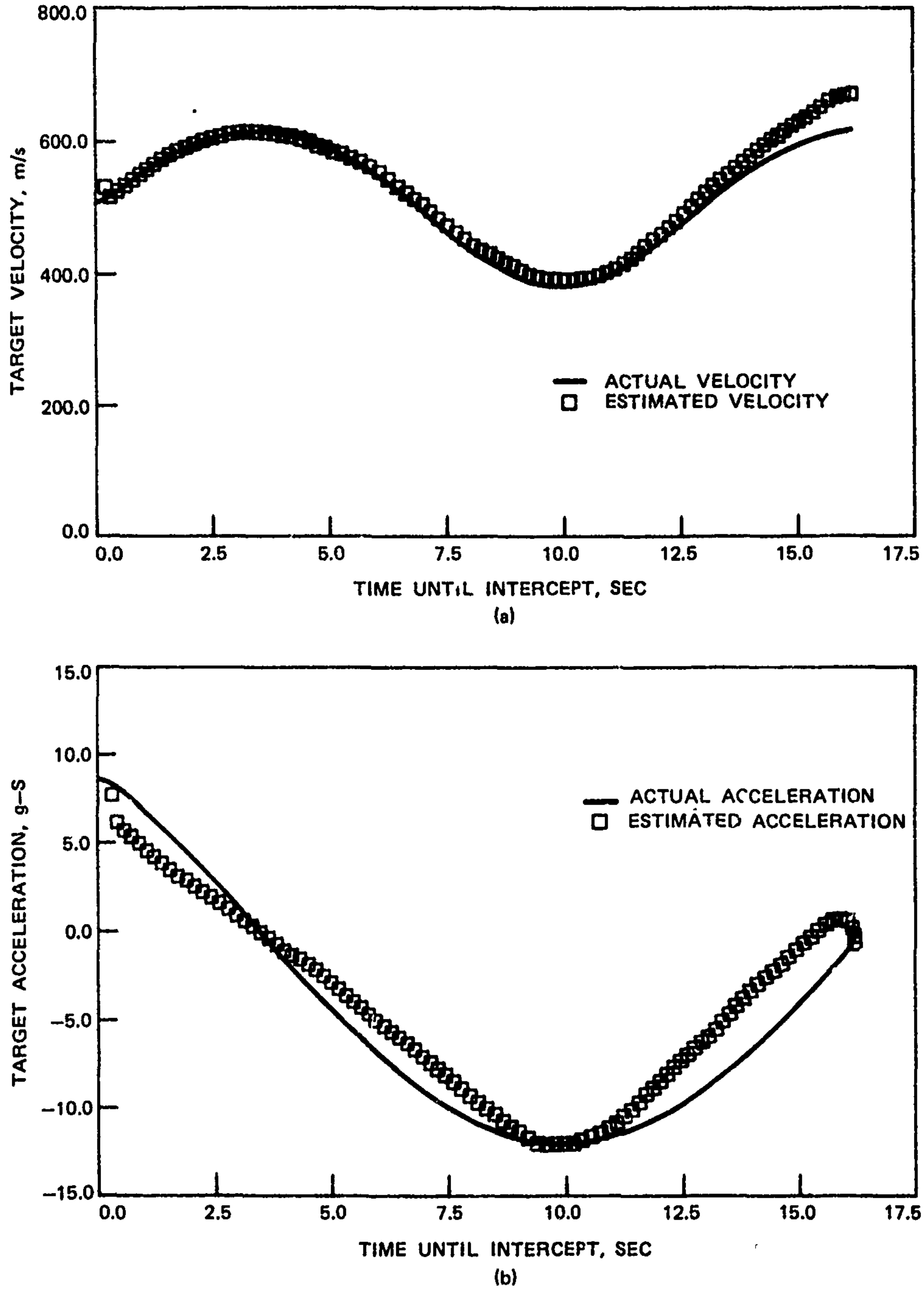


FIGURE 19. Comparison Between Estimated and Actual Target Acceleration and Velocity for Example 5 (Sampling interval $\Delta t = 0.01$ sec).

SUMMARY AND CONCLUSIONS

In this report a simple algorithm is presented for estimating target velocity and acceleration. The algorithm uses as inputs the measurement of range, range rate, line-of-sight angle, line-of-sight rate, and missile velocity. The estimated target acceleration is then included in a missile guidance law and the effectiveness of such a missile compared to a similar one using proportional navigation.

The primary advantage of a missile using a target estimator is that it provides a more direct (shorter) trajectory between missile and target due to its ability to correct for target maneuvers more quickly. When proportional navigation is employed, there is a tendency for the missile to be acceleration limited (i.e., pulling maximum g's) near the end of its trajectory as the missile tries to compensate for the maneuver. The use of the target acceleration estimator in the guidance law provides a means of correcting for the target maneuver more quickly and thus fewer target misses due to missile g-limiting. This would seem to be of particular advantage at high altitudes where missile maneuverability is limited and airframe response is slow (see Figure 13). An additional advantage of the use of target acceleration estimation in the guidance law is a slightly shorter time of flight (0.2 to 0.5 sec for the examples considered in this report) until intercept with the target. The price to be paid for target acceleration estimation is in terms of additional sensors and system complexity. A comparison of Equations 40, 41, and the target acceleration estimator as given by Figure 5 reveals that in order to estimate target acceleration the additional variables (i.e., those variables in addition to closing velocity and line-of-sight rate used in proportional navigation) that need to be measured are range, line-of-sight angle, and missile velocity. In addition, a small computer is necessary in order to implement the acceleration estimator as diagrammed in Figure 5.

The estimation algorithm presented here appears to provide a fairly good estimate of target acceleration for the noise-free cases considered. In the actual missile one can expect incorrect measurements of range, range rate, level-of-sight rate, etc., needed to implement the algorithm presented in this report. These measurement errors can result from glint, clutter, receiver noise, eclipsing, countermeasures, etc. The sensitivity of the estimator to these noise sources was beyond the scope of this study.

Only one guidance law (i.e., the one given by Equation 41) was investigated in this study. Also, the estimator given here appears to estimate target velocity more accurately than target acceleration (see Figure 17 for example). Unfortunately, there seems to be little in the way of guidance laws based on target velocity rather than on target acceleration. As mentioned previously, the weighting factor of 1/2 appearing in Equation 41 results from optimal control theory based on simplifying assumptions. It is not known how sensitive terminal distance is to this parameter. The method of switching the target acceleration estimator out of the guidance law (Equations 41 and 42) produces an abrupt change in the commanded acceleration (see Figure 18 for example). It may be more desirable to gradually switch the acceleration out of the guidance law than to use Equation 42. In an analog circuit a "soft" switch can be implemented using a low-pass filter in conjunction with Equation 42. Digitally, it can be done by replacing Equation 42 with

$$S(t_{go}) = \frac{t_{go}^n}{1 + rt_{go}^n}$$

where n and r are positive numbers which can be chosen to provide a less abrupt switch than in Equation 42.

Lastly, the estimator given in this report was derived under the assumption of target and missile coplanar throughout the endgame. While it is felt that the generalization from two to three dimensions would be straightforward, this has not been done at this time but would be necessary if such an estimator were to be actually implemented in a missile.

While this report is mainly concerned with using a target velocity and acceleration estimator in a missile, another possible application would be in the launch aircraft itself as a missile fire control indicator. A proposed method for indicating when the missile should be launched uses precomputed launch regions (launch envelopes) that indicate to the pilot that the geometry relating target and missile is such that the missile has a good chance of intercepting the target. Whether or not a missile can successfully interrupt a target depends to a large degree on target velocity and acceleration. The use of a target acceleration and velocity estimator such as the one proposed in this report could be used to generalize the launch aircraft fire controller to include the case of maneuvering targets.

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